Empirical Symmetries

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Introduction

Recent discussions of symmetry in physics have focused on symmetries of physical *theories* of various sorts (dynamical, space-time, gauge etc.) Sometimes a symmetry becomes apparent only after examination of the structure of an existing theory. But in other cases the recognition of a symmetry in nature has stimulated construction of a theory to explain that symmetry.

To understand this and other roles of symmetry in physics we need a notion of *empirical* symmetry to contrast with any sort of symmetry defined in terms of transformations among the laws, equations or models of a theory. I shall offer an analysis of such a notion and show what it is good for, in metaphysics as well as the philosophy of science. Armed with this analysis, we shall be able to…

…understand Time-Reversal,

- How “sophisticated” space-time substantivalists can agree with everyone else that
- evidence for a time-reversal invariant theory can undermine one good reason for distinguishing past and future directions of time; and that
- time-reversal has (only) an active interpretation (though not even Superman can reverse time!)

appreciate Einstein's insights,

- Why, in developing each of his theories of special and general relativity, Einstein's key insight was his recognition that an empirical symmetry is not merely strong but perfect

and understand gauge symmetry

- Why Faraday’s cube, despite its analogy to Galileo’s ship, does *not* show that a gauge transformation reflects an empirical symmetry in our world; though there is a nearby possible world in which it *does* show this

Symmetries of a Structure

Consider a structure $\mathcal{S} = \langle D, R_1, R_2, \ldots, R_n \rangle$.

Let f: $D \rightarrow D$ be a 1-1 mapping of the domain of $\mathcal{S}$ onto itself.

Define the transformed structure $\mathcal{S}_f$ by

$\mathcal{S}_f$ _ $\langle D, f^*R_1, f^*R_2, \ldots, f^*R_n \rangle$, where

$f^*R_i\,[f(d_1), \ldots, f(d_m)]$ if and only if $R_i\,[d_1, \ldots, d_m]$

for an m-place relation R_i.

Then f is a *symmetry of $\mathcal{S}$* just in case $\mathcal{S}_f = \mathcal{S}$.

Examples

A snowflake is an object with a symmetric structure.

One symmetry transformation corresponds to a clockwise rotation through 60^o: this maps each crystal in one “arm” onto a similar crystal in another “arm”, while preserving the appearance of the snowflake.

Newtonian space-time is an object with a symmetric structure.

One symmetry transformation is a mapping of each space-time point onto a point related to it by a “temporal reflection” about an arbitrarily chosen instant (a simultaneity hyperplane).

Symmetries of a physical object are empirical

For a physical object, the elements of *D* are parts of the object.

One can observe and measure the object to collect evidence that relations among its parts are or would be indifferent to the action of a symmetry transformation of its parts.

Such evidence may be direct (the snowflake) or indirect (Newtonian space-time).

Symmetries among Situations

Situations of a certain kind *K* may all have a similar structure. Any situation with that structure may be transformed into another by a transformation *f*. If a subset *D* of *K* is closed under *f*, then *f* is a symmetry of the “larger” structure _ = $< D,P_j >$, where the properties P_j define the kind *K*.

(e.g. *K* may be situations represented by kinematically possible models of a theory, while those in *D* are represented by dynamically possible models.)

Examples

Renderings of a tune all have a similar structure, even if they are in different keys and some include errors. Flawless renderings of the same tune are related by a symmetry transformation that transposes one key into another.

The processes Galileo describes as occurring in the cabin of a ship have exactly the same dynamic structure, independent of how fast the ship moves over the sea. Different instances of each process are related by the same symmetry transformation—corresponding to a change in the uniform horizontal velocity of the ship.

Empirical symmetries among situations

One can observe and measure situations to collect evidence that situations related by a symmetry transformation cannot be distinguished by specific procedures.

Flawless renderings of a tune in different keys can be directly distinguished by someone with perfect pitch: the rest of us may need instruments.

According to Galileo, measurements of purely mechanical magnitudes inside the cabin cannot distinguish between different states of uniform horizontal motion of a ship.

Empirical Symmetries

An empirical symmetry is a feature of a class of situations—actual as well as possible.

A 1-1 mapping _: **S** _ **S** of a set of situations onto itself is an *empirical symmetry with respect to C-type measurements* if and only if no two situations related by _ can be distinguished by measurements of type *C*.

Strong Empirical Symmetries

A 1-1 mapping _: **S** _ **S** of a set of situations onto itself is a *strong empirical symmetry* if and only if no two situations related by _ can be distinguished by measurements confined to each situation:

A measurement is *confined to a situation* if and only if it is a measurement of intrinsic properties of (one or more objects in) that situation.

Perfect Empirical Symmetries

An empirical symmetry _ is *perfect* if and only if any two situations related by _ are duplicates:

A *duplicate* of a situation is a situation that shares all its intrinsic properties.

A theory may account for an empirical symmetry

The hexagonal symmetry of a snowflake may be explained by a theory of crystal structure and formation.

The empirical symmetry noted by Galileo may be explained by a theory of dynamics such as Newton’s or Einstein’s.

The account often appeals to a symmetry of the theory

The dynamics of Newton's theory are symmetric under uniform velocity transformations—Galilean boosts.

The dynamics of Einstein's theory are symmetric under uniform velocity transformations—Lorentz boosts.

A theory uses models to represent situations

In the case of a dynamical theory, each model requires the dynamical history of a situation it represents to conform to its dynamical laws—typically stated as equations of motion.

Since the statement of dynamical laws presupposes some space-time structure, the models must also represent this.

Theoretical Symmetries

A theoretical symmetry is a feature of the models of a theory.

A 1-1 mapping *f*: **M** _ **M** of the set of models of a theory _ onto itself is a *theoretical symmetry* of _ if and only if the following condition obtains:

For any model *m* of _ that may be used to represent (a situation *s* in) a possible world *w*, *f*(*m*) may also be used to represent (*s* in) *w*.

Symmetries in dynamical theories

Suppose we believe a dynamical theory _, whose models represent various goings-on in space-time. Then theoretical symmetries of _ can inform us about symmetries of our space-time.

John Earman talks about this in terms of *dynamical symmetries* and *space-time symmetries* of a theory. He argues that the dynamical symmetries of any well-formulated theory are the same as its space-time symmetries.

This implies that every theoretical symmetry of a well-formulated theory corresponds to a symmetry of the space-time it represents. In more detail…

Models of a dynamical theory

"The intended models of a classical theory of motion have the form

<$M, A_1, A_2, \ldots, P_1, P_2, \ldots$>, where the absolute objects A_i are geometric object fields characterizing the fixed space-time structure and the dynamic objects P_j are geometric object fields characterizing the physical contents of space-time." (Earman, *World Enough and Space-Time*, p.45)

Dynamical Symmetries

“Consider a model $m = <M,A_1,A_2,\ldots,P_1,P_2,\ldots>$ and let h be a diffeomorphism that maps M onto M.

Define m_g _ $<M,A_1,A_2,\ldots,g^*P_1,g^*P_2,\ldots>$. Now g will be said to be a *dynamic[al] symmetry* of _ just in case for any model m of _, m_g is also a model of _.”

(Earman, *ibid.)*

i.e. g necessarily preserves the truth of _’s dynamical laws

Earman's Condition

Earman has argued that any well-formulated dynamical theory _ should satisfy the following condition:

The dynamical symmetries of _ are the same as the space-time symmetries of _.

This should seem puzzling. A space-time symmetry is a feature of a physical object posited by _, while a dynamical symmetry is a feature of a class of _'s models.

To solve the puzzle, see how Earman's definition of space-time symmetry relates to the general notion of a symmetry of a physical object.

Space-time Symmetries

"A *space-time symmetry* of the fixed space-time is a mapping that leaves all the A_i invariant, i.e., a diffeomorphism *h* that maps *M* onto *M* in a way that $h^*A_i = A_i$, for all *i*."

(Earman, *ibid.*)

Call such a diffeomorphism *h* an *Earman s-t symmetry*.

Why Earman s-t symmetries preserve space-time structure

Suppose a fixed space-time *S* is an object with structure $\mathcal{S}$ = <*D*,R_1,R_2,…,R_n>, where *D* is a set of space-time points, and the R_j are intrinsic relations among these.

A model *m* = <*M*,A_1,A_2,…> represents this structure, where *M* is a differentiable manifold whose base-points represent space-time points, and whose absolute objects A_i represent intrinsic spatio-temporal relations.

A manifold diffeomorphism *h* is a smooth 1-1 mapping of the base-points of *M* onto themselves.

h: *M* _*M* yields a model *m** = <*M*, h^*A_1, h^*A_2,…> that also represents a space-time *S**:

Under the same representational conventions, *S** has structure

$\mathcal{S}_f$ _ <*D*, $f_h{}^*R_1$, $f_h{}^*R_2$,…, $f_h{}^*R_n$>, where the map f_h on *D* is represented by the map *h* on *M*.

h is an Earman s-t symmetry if and only if *m** = *m*. But this is true if and only if $\mathcal{S}_f$ = $\mathcal{S}$, i.e. if and only if *f* is a symmetry of the structure $\mathcal{S}$.

Relation between dynamical and space-time symmetries

As Earman shows, every Earman s-t symmetry is a dynamical symmetry: hence every space-time symmetry entails a corresponding dynamical symmetry.

Similarly, in a well-formulated theory, every dynamical symmetry entails a corresponding space-time symmetry.

(A theory is well-formulated if it contains no structure that is superfluous since it is not needed to formulate its laws of motion.)

Time-Symmetric Space-times

Time reversal is a symmetry of a fixed (globally hyperbolic) space-time *S* if and only if, for every instant *t*, the function that "reflects" every point of *S* in *t* is a symmetry of *S*'s space-time structure.

Call such a space-time *time symmetric*.

If *our* space-time is time-symmetric, there is no intrinsic difference between the past and future directions of time!

Time-reversal Invariant Theories

If _ is a well-formulated dynamical theory of which time-reversal is a dynamical symmetry, the space-time postulated by _ is time-symmetric.

Time-reversal is a dynamical symmetry of _ just in case

for any model *m* of _, m_T is also a model of _, where $m = <M, A_1, A_2, \ldots, P_1, P_2, \ldots>$

$m_t = <M, A_1, A_2, \ldots, T_t{*}P_1, T_t{*}P_2, \ldots>$, and T_t represents an arbitrary time-reversal function.

Testing Time-Symmetry

How can we decide whether there is any intrinsic difference between past and future time directions?

We can't directly test the claim

N_oD_{ifference}: *Space-time is time-symmetric*,

since we can't directly observe space-time points!

But we *can* check whether time-reversal invariance is an empirical symmetry of all situations modeled by our fundamental theories.

Experiments could support ND!

Suppose all our fundamental theories are time-reversal invariant, i.e. time-reversal is a dynamical symmetry of all these theories. Suppose further that these theories are all well-formulated. Then they all agree that our space-time is time-symmetric. Moreover, the time-reverse of any situation modeled by these theories will also be modeled by them.

We can therefore acquire evidence for claim *ND* by measuring all relevant quantities in situations related by a time-reversal transformation. So we can verify that time-reversal is an empirical symmetry of those situations, and confirm that our theories successfully model these situations *because* time-reversal invariance is a dynamical symmetry of those theories.

Our theories may be incomplete, and there may be some situation they cannot model, whose time-reverse is incompatible with some *true*, non-time-reversal invariant theory. But an exhaustive search could render this increasingly unlikely.

A refutation of space-time substantivalism?

Brad Skow has argued that a “sophisticated” space-time substantivalist cannot accept an argument from the time-reversal invariance of Newtonian mechanics to the conclusion that Newtonian space-time supports *ND*.

We can now see why this argument fails.

A space-time substantivalist maintains that space-time exists, over and above its material contents.

In this context, what makes him “sophisticated” is the further claim that the material contents of space-time could not all have been “temporally inverted” with respect to some instant of a time-symmetric space-time.

Skow's Argument

His argument is very simple. The "sophisticated" space-time substantivalist cannot accept an argument for the conclusion that Newtonian space-time is time-symmetric based on a premise that Newtonian mechanics is *TRI* since he is committed to believing that the conclusion is inconsistent with the premise!

To show that this is so, Skow takes the claim that Newtonian mechanics is *TRI* to imply for a Newtonian world (only) that it is possible that all particle trajectories be the reverse of what they actually are in that world.

He then argues that if Newtonian space-time is time-symmetric, then the "sophisticated" substantivalist must reject this possibility.

The Flaw in the Argument

But the claim that Newtonian mechanics is *TRI* does *not* imply that in a Newtonian world it is possible that all particle trajectories be the reverse of what they actually are in that world.

TRI is a dynamical symmetry of Newtonian mechanics. This means that for any model m of Newtonian mechanics, m_t is also a model, where $m = \langle M, A_1, A_2, \ldots, P_1, P_2, \ldots \rangle$, $m_t = \langle M, A_1, A_2, \ldots, T_t{*}P_1, T_t{*}P_2, \ldots \rangle$, and T_t represents an arbitrary time-reversal function.

A Newtonian world *w* has a *global* model *Gm.*

It follows that Gm_t is *also* a global model of a Newtonian world *w**: But which one?

Skow assumes that $w^*=w_t$—a world in which all particle trajectories are the reverse of what they are in *w*. But of course the “sophisticated” substantivalist denies this, claiming rather that $w^*=w$! There is no inconsistency in this position. The “sophisticated” substantivalist is free to use the evidence that *TRI* is an empirical symmetry of *non-global* situations in *w* to argue for the time-symmetry of *w*.

An Insight Remains

But note that if a theory can *only* model global situations, then one *cannot* get evidence that it is *TRI* by observing local empirical time-symmetries. That is as it should be, for the claim that such a theory is *TRI* becomes trivial. *Every* purely global theory that admits a “time-reflection” operation is trivially *TRI* in its application to the actual world: the merely possible worlds can look after themselves!

Is time-reversal an active or passive transformation?

Most empirical space-time symmetries clearly have active interpretations. One can move the apparatus around in space, wait and do the experiment later, or put it in a plane or space-ship and do it at a different velocity. But not even superman can reverse time so the experiment runs backward!

Most empirical space-time symmetries also have passive interpretations, in which one thinks of leaving the apparatus alone and changing one's coordinate system or reference frame. Moreover this passive interpretation is typically claimed to be equivalent to the active interpretation.

It seems plausible that time-reversal has only a passive interpretation involving switching the t coordinate with $-t$.

…plausible, but Wrong!

I think I remember Michael Redhead saying once that time-reversal has only an *active* interpretation. I immediately thought that he was wrong, but I continued to wonder why he would have made this mistake.

Now I want to show you why he was right!

What *are* active and passive transformations?

The key point to understand is that while a *passive* interpretation of a transformation does involve a change of representation (coordinate system, frame,…) rather than an operation on the system represented, this change is uniquely correlated with an operation on systems. Every transformation with a passive interpretation also has an active interpretation, but not *vice versa*. It is a mistake to assume that every change of representation corresponds to a passive transformation.

Representing the Observer

It is always important to distinguish between *representations* (models, equations in a particular coordinate system, frame-dependent magnitudes) and *what they represent* (systems, situations, dynamical trajectories, invariant magnitudes).

On an active interpretation, a transformation T relates actual or possible physical *situations*: this has a passive interpretation only if T corresponds uniquely to a transformation T_R that relates *representations* of such physical situations.

The correspondence may be set up only on two conditions:

1) The situation of a possible observer may be represented in the same way as other physical situations related by T, and

2) The transformation T may be applied to the situation of a possible observer to produce a situation of a possible observer.

Active Transformations

An active transformation is a function T:$\mathbf{S} \rightarrow \mathbf{S}$ from a set of physical situations onto itself.

If the two conditions are satisfied, then it is meaningful to include the situation O of a possible observer in **S** and to apply T to O, resulting in a situation $O^* = T(O)$ of a possible observer.

Passive Transformations

A *passive transformation* is a function T_R: $\mathbf{R} \rightarrow \mathbf{R}$ from a set of representations of physical situations onto itself.

But not every function that yields a change of representation counts as a passive transformation!

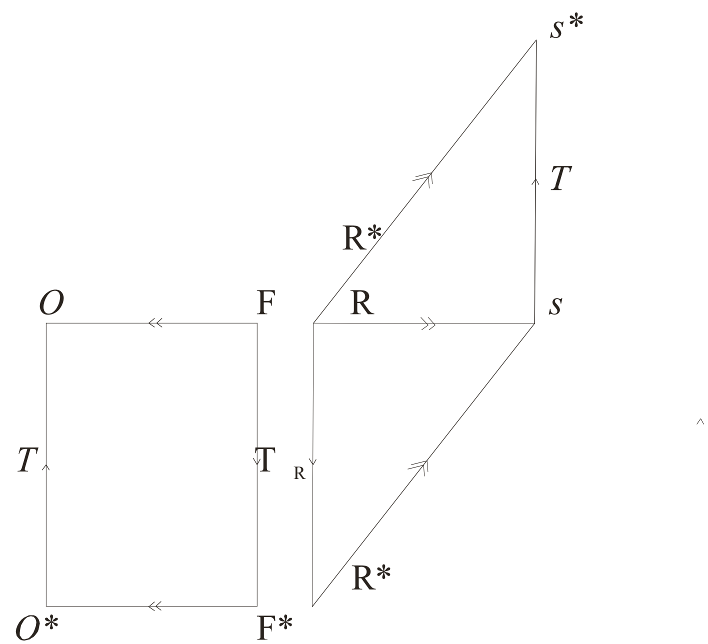
Representations and Frames

It is natural to call a representation of the situation of a possible observer a *frame*.

An observer *O* in a situation represented by a frame F will have his own representations R,R* of other physical situations.

We can now explain the difference between active and passive interpretations of a transformation.

Passive Transformation



The transformation T_R of representations of situations is a ***passive transformation*** of a representation R of a situation *s* if and only if, when applied to frame F representing the situation of observer *O*, it produces a frame F* representing that of an observer *O**, where *O**’s representation R* of *s** in F* is the same as *O*’s representation of *s* in F, and the situations of *s* and *s** are related by the same (active) transformation *T* as *s* and *s**.

Time-Reversal has only an active interpretation

In order for time-reversal to have a passive interpretation, the two conditions would have to be satisfied. But only the first condition is met.

The situation of an observer *may* be represented in a model of a space-time theory, e.g. by an oriented time-like curve.

But applying the time-reversal operation to this model, e.g. by *reversing* the orientation of this curve, does *not* yield a representation of a possible observer.

So no passive interpretation is possible.

How is time-reversal possible?

But how can time-reversal involve an active transformation from one physical situation to another if one can't actively reverse the direction of time?

To realize the transformation it is not necessary to attempt this impossible feat. All that is required is to find or construct a process similar in all relevant respects to a given process, but whose initial state is the time-reverse of the final state of the original process. One way to do that is to manipulate the original process, e.g. by reversing the directions of all the particles involved or by flipping the spins of all atoms in a spin-echo experiment. But the time-reversed situation need not involve the original system, but merely a replica of it. No physical manipulation of a system involved in the original situation is required: one can realize an active transformation of a given situation without touching anything involved in that situation.

How a theory explains a perfect symmetry

Suppose T_R is a theoretical symmetry of theory _, where $R^*= T_R(R)$, and the models R,R* offer equivalent theoretical representations of some situation *s*.

Suppose further that R represents *s* from the perspective of some possible observer *O*, in the following sense: If a representation F of *O* were added to R, F would occupy some privileged role in the representation. (e.g. R may be a representation of *s* in an inertial coordinate system that represents the world-line of an inertially moving observer by a directed line F at its spatial origin).

It may be that one can add to R* a representation F* that occupies precisely the same role in R* that F plays in R. Call what F* represents *O**. There is a unique transformation *T* that maps *O** onto *O*. Let *T*(*s*)=*s**. Then one can add to R a representation of *s**, and the resulting representation will represent *s** in exactly the same way that R* represents *s*.

Hence, as far as _ is concerned, *s* and *s** are perfect duplicates—they share all intrinsic properties _ is capable of representing!

Einstein’s First Insight

Einstein began his first paper on special relativity by focusing on strong empirical symmetries in electro-dynamic phenomena associated with different states of uniform motion. These could be explained by the theory of electromagnetism as then understood. But the explanation did not appeal to a corresponding dynamical symmetry of the theory: its models were not related by a velocity boost.

In response, he revised our understanding of electromagnetic theory by resetting it within a new space-time structure. The empirical symmetries he focused on could now be understood as flowing from dynamical symmetries of the theory. Because that theory was well formulated, a uniform velocity boost could now be recognized as a perfect empirical symmetry of these electro-dynamic phenomena, and indeed of all other non-gravitational situations.

Einstein’s Second Insight

On the road to general relativity, Einstein focused on a particular empirical symmetry of gravitational phenomena that followed from the equality of (passive) gravitational and inertial mass.

Measurements of mechanical phenomena cannot distinguish a situation involving a uniform gravitational field from a similar situation involving a uniform acceleration. Consequently,

“The gravitational field has only a relative existence... Because for an observer freely falling from the roof of a house - *at least in his immediate surroundings* - there exists no gravitational field.”

Einstein called this “the happiest thought of my life”!

His general theory of relativity finally enabled him to explain this empirical symmetry as an instance of a perfect empirical symmetry: anything experiencing a uniform gravitational field *is* simply accelerating with respect to the local inertial structure of space-time.

Concluding Morals

The notion of an empirical symmetry needs to be understood without reference to the equations or models of any theory.

One can acquire evidence for an empirical symmetry without a theory to account for it: but symmetries of a theory can provide satisfying explanations of empirical symmetries.

Most, though not all, empirical symmetries relate *local* situations rather than global "worlds".