A Tale of Two Symmetries:

or,

is Faraday in the same boat as Galileo?

Galileo's ship

Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though there is no doubt that when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still.

Faraday’s Cube

“I went into this cube and lived in it, but though I used lighted candles, electrometers, and all other tests of electrical states, I could not find the least influence on them”

(from his1836 diary entry)

The puzzle

Both Faraday and Galileo are describing observations of symmetries in nature.

In each case, different situations are compared, and it is noted that these are indistinguishable with respect to a whole class of phenomena.

But while Galilean invariance reflects a paradigm empirical symmetry, gauge symmetry is usually thought of as a purely formal feature of a theory. In this case, adding the same constant to all electrical potentials is a gauge symmetry of classical electromagnetism.

Why doesn't Faraday's cube provide a *perfect* analogue of Galileo's ship for gauge symmetry?

A Relativity Principle

- A relativity principle is associated with Galilean/Lorentz invariance:
- *situations related by a transformation from one state of uniform motion to another are internally indistinguishable*.
- A relative velocity transformation is a *symmetry* of such situations.

Another Relativity Principle?

- Is this another relativity principle associated with gauge invariance:
- *situations related by a transformation from one state of uniform electric potential to another are internally indistinguishable*?
- Is a relative electric potential transformation a *symmetry* of such situations?

Is gauge symmetry empirical?

No: despite the seductive analogy between Galileo’s ship and Faraday’s cube, there is a critical disanalogy.

Gauge symmetry remains a purely formal feature of classical electromagnetism.

But Faraday’s cube exhibits a *different* empirical symmetry associated with that theory,

And there is a nearby possible world in which gauge symmetry *is* an empirical symmetry, on a par with Galilean invariance.

What counts as observing a symmetry?

Kosso(2000) stated two necessary conditions for the observation of a symmetry of interest to physics:

1) One must observe that the specified transformation has taken place
2) One must observe that the specified invariant property is in fact the same, before and after

Empirical symmetries need not be observable…

An empirical symmetry may or may not be observable: it may be too hard to create the necessary situations, or to find them realized in nature.

What makes a symmetry empirical is just that the necessary measurements would reveal it *if* they could be performed in actual situations.

…but

one may be able to observe an empirical symmetry whether or not one has a theory that accounts for it.

What *is* an empirical symmetry?

An empirical symmetry is a feature of a class of situations—actual as well as possible.

A 1-1 mapping _: **S** _ **S** of a set of situations onto itself is an *empirical symmetry with respect to C-type measurements* if and only if no two situations related by _ can be distinguished by measurements of type *C*.

Strong empirical symmetries

A 1-1 mapping _: **S** _ **S** of a set of situations onto itself is a *strong empirical symmetry* if and only if no two situations related by _ can be distinguished by measurements confined to each situation:

A measurement is *confined to a situation* if and only if it is a measurement of intrinsic properties of (one or more objects in) that situation.

A theory may entail an empirical symmetry

Galileo's ship illustrates an empirical symmetry (loosely) associated with uniform velocity transformations.

This empirical symmetry is entailed by Newton's theory of motion, assuming that all forces and masses are independent of absolute velocities. For then no measurement of purely mechanical properties of systems in the cabin can distinguish between different states of uniform motion of the ship.

A theory may entail a strong empirical symmetry…

Special relativity entails a strong empirical symmetry associated with uniform velocity transformations (Lorentz boosts).

...or even a perfect empirical symmetry

The Lorentz boost symmetry entailed by the theory of special relativity is not just strong but perfect.

An empirical symmetry _ is *perfect* if and only if any two situations related by _ are duplicates:

A *duplicate* of a situation is a situation that shares all its intrinsic properties.

Theoretical Symmetries

A theoretical symmetry is a feature of the models of a theory.

A 1-1 mapping *f*: **M** _ **M** of the set of models of a theory _ onto itself is a *theoretical symmetry* of _ if and only if the following condition obtains:

For any model *m* of _ that may be used to represent (a situation *s* in) a possible world *w*, *f*(*m*) may also be used to represent (*s* in) *w*.

A theoretical symmetry may entail an empirical symmetry

If a model of special relativity incorporates a reference frame, then the Lorentz boost Λ_{-v}(*m*) of model *m* may be used to represent the very same situation *s* as *m*.

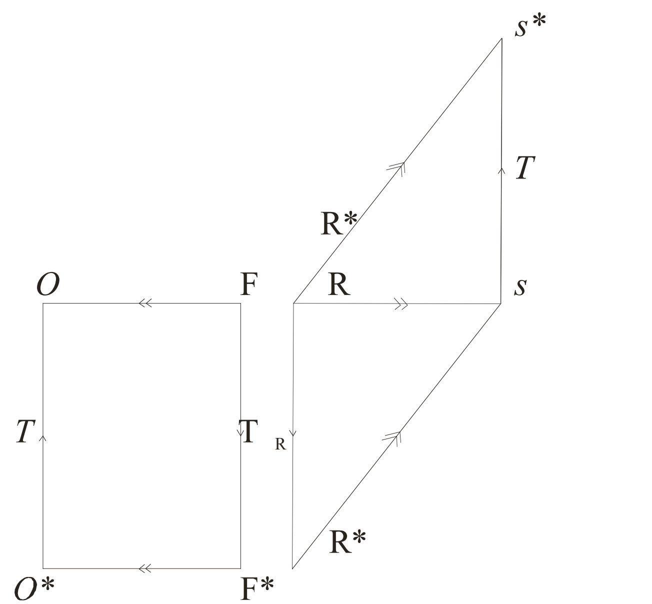
But *m* may alternatively be used to represent the boosted situation Λ_v(*s*).

And *m*'s representation of Λ_v(*s*) is then identical to Λ_{-v}(*m*)'s representation of *s*.

So the theory entails that *s* could not be distinguished from Λ_v(*s*) by means of *any* properties within its scope, including any that may be revealed by measurements confined to each of those situations.

Lorentz invariance as an empirical symmetry follows from a theoretical symmetry of special relativity.

How this works



Suppose the transformation T_R of representations of situations is a theoretical symmetry that maps one representation R of a situation *s* into another representation R*. Applied to frame F representing the situation of observer *O*, it produces a frame F* representing the situation of observer *O**, where *O**'s representation R* of is the same as *O*'s representation of a situation *s* such that are *s s** and related by the same (active) transformation *T* as . Then *O** and *O*
T is a perfect empirical symmetry. *O** presents with a situation that is a duplicate of the situation with which *s* presents . *O**

But a theory can entail an empirical symmetry in another way

We’ve seen that Newton’s theory entails the empirical symmetry that all mechanical processes go on in the same way irrespective of state of uniform motion.

But this is *not* a consequence of a theoretical symmetry of Newton’s theory:

Newton’s theory represents different states of uniform motion by different models, and so itself distinguishes among them.

Gauge symmetry is a theoretical symmetry

The transformation $\phi \rightarrow \phi + a$ is a theoretical symmetry of models of electromagnetism, for arbitrary constant a.

This is a special case of the gauge transformation $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$, where $A_\mu = (\phi, -\boldsymbol{A})$ and $\Lambda = at$

But does this theoretical symmetry entail the empirical symmetry Faraday observed in his cube?

Gauge symmetry does not entail a corresponding empirical symmetry

At first sight, it seems to.

The situation in the charged cube seems related to the situation in the uncharged cube by the transformation $\phi \rightarrow \phi$ + _ϕ with ϕ,_ϕ constants.

But this is too quick: rather, if the situation in the uncharged cube is represented by a model *m* with electric potential ϕ, then the situation in the charged cube is represented by a model *m′* with electric potential ϕ + _ϕ.

Why not?

The theoretical gauge symmetry implies that *m*,*m′* may be used to represent the very same situation.

But we only have an empirical symmetry here if *m*,*m′* may *also* be used to represent distinct but indistinguishable situations *s*,*s′* inside the cube, themselves related by a physical gauge transformation.

But isn't this just like Lorentz symmetry?

Why doesn't the argument from theoretical to empirical symmetry go through in this case, while it does in the case of Lorentz symmetry?

It goes through in the case of Lorentz symmetry because special relativity implies not only that situations *s* and $\Lambda_v(s)$ are theoretically indistinguishable, *but also that they are related by the transformation* Λ_v.

For *every* joint model of the combined situation *s* + $\Lambda_v(s)$ represents *s* and $\Lambda_v(s)$ as related by the velocity boost Λ_v.

Electric fields need not arise from differences in electric potential

Electromagnetic theory contains joint models of the combined situation (uncharged cube) + (charged cube) that represent the cube's interior as differing only in electric potential by the constant _ϕ.

But it *also* contains models that represent its interior as at the *same* electric potential, whether or not it is charged.

Models of the latter variety represent the electric field outside the charged cube as due in part to the time-variation in a *magnetic* vector potential (curl-free, so there is no magnetic field).

So charging Faraday's cube need not change its potential

The theoretical gauge symmetry of electromagnetic theory entails no corresponding empirical gauge symmetry in situations involving different charge states of Faraday's cube.

It implies no such empirical symmetry because it does not imply that there are distinct situations represented by *m*,*m' that are related by a physical difference* $\_\phi$ *in electric potential*.

Gauge transformations are not empirical

It is natural to describe the state of Faraday's cube when charged by saying that it has been raised to an electric potential with respect to the ground.

But this is not something that we *observe*—all we observe are differences in electric field outside the cube when charged and uncharged.

And it is not entailed by electromagnetic *theory*, since that theory has models that represent this electric field as arising not from a difference in electric potential, but from variations in magnetic vector potential.

But Faraday's cube *does* exhibit a *different* empirical symmetry

Electromagnetic theory implies that charging Faraday's cube gives rise to a different situation; since *all* models of that theory imply that the electromagnetic field outside the cube depends on whether or not it is charged.

But this theory also implies that the electromagnetic situation *inside* the cube is *independent* of whether or not it is charged.

So electromagnetic theory entails a *perfect* empirical symmetry here.

But this is *not* a consequence of gauge symmetry, which remains a purely theoretical symmetry.

And gauge symmetry might have been empirical!

In a world with static electricity but no magnetism, the empirical success of electrostatics might have justified belief in the empirical adequacy of a theory in which the *only* joint models of the combined situation (uncharged cube)+(charged cube) represented an electric potential difference between the cubes of the same constant value $\_\phi$.

Gauge symmetry *is* empirical in nearby worlds

In such a world, the limited theoretical gauge symmetry $\phi \rightarrow \phi + a$ *would* have implied a corresponding empirical gauge symmetry. We would have had indirect empirical evidence that differences in electric potential are real, as are transformations from one state of electric potential to a state of lower or higher potential—just as, in our world, we have (both direct and) indirect evidence that differences in uniform velocity are real, as are transformations from one uniform velocity to another.

Such a world is not very distant from ours.